



Communications Theory

Introduction

Manuel A. Vázquez

January 29, 2024

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Instructors

Manuel A. Vázquez

- 4.3.A02B
- mavazque@ing.uc3m.es

(Theory + Exercises)

Sipon Miah

- 4.2.D09
- mdsiponmiah@tsc.uc3m.es

(Lab sessions)

Material

To be found in **Aula Global**:

- Ad-hoc notes

Where the decision rule matters

Notice that the decision rule is only required to hold for the decision region.

2.14 Computation of the probability of error when $N > 1$

We can just group them in a single of the nearest neighbor.

2.14.1 If $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M \in \mathcal{R}^N$, Gaussian vector

This is not the visualization for which we have computed the error energy and distance. The corresponding signals here is one and a vector (they are orthogonal). We follow the next step.

We compute the decision region: the optimal ML decision is guaranteed because the signals are equally likely and the noise is Gaussian. However, we can give the conditional pdf's together to see what's going on. There the optimal rule is guaranteed, in order to get the decision region, we look that the conditional probability of the signal \mathbf{x}_1 is the highest. The probability of the signal \mathbf{x}_1 is the highest and since it is higher than any other, and also since the noise can be Gaussian pdf, we have the decision rule as we working from the noise in the region.

the probability of error is written in terms of the conditional probability of error

$$P_e = P_{e1|1} + P_{e2|2}$$

We compute the conditional probability distribution. From the energy E_{s1} and the probability of the conditional error $P_{e1|1}$ and the error $P_{e2|2}$ is

$$P_{e1|1} = \int_{\mathcal{R}_1^c} p(\mathbf{r} | \mathbf{x}_1) d\mathbf{r} = \int_{\mathcal{R}_1^c} \frac{1}{(2\pi)^{N/2}} e^{-\frac{1}{2}(\mathbf{r} - \mathbf{x}_1)^T (\mathbf{r} - \mathbf{x}_1)} d\mathbf{r}$$

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X and Y dependent

X and Y independent

Joint entropy in the union of the sets

The joint entropy $H(X, Y)$ is the union of the intersection of X and Y (denote the last that the conditional pdf's given the distribution of all the probabilities of X and Y).

From the formula that relates conditional and joint entropy we can write that as the given to the left and right of the intersection.

if we remove $H(Y)$ from the union,

$$H(X, Y) - H(Y) = H(X|Y)$$

and if we remove $H(X)$ from the union,

$$H(X, Y) - H(X) = H(Y|X)$$

Notice that we still don't have a source for the intersection.

2.3.1 Parabolic channel model

We can apply the optimization to each part function and parabolic channel model. If it is the input \mathbf{Y} of the system

a R, X and Y are independent

$$H(X, Y) = H(X) + H(Y)$$

that is, knowing Y does not provide any information about X , which means the uncertainty in their about X is the same as had before knowing Y . In other words, the transmission rate is fixed.

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- Slides

Planning of the course

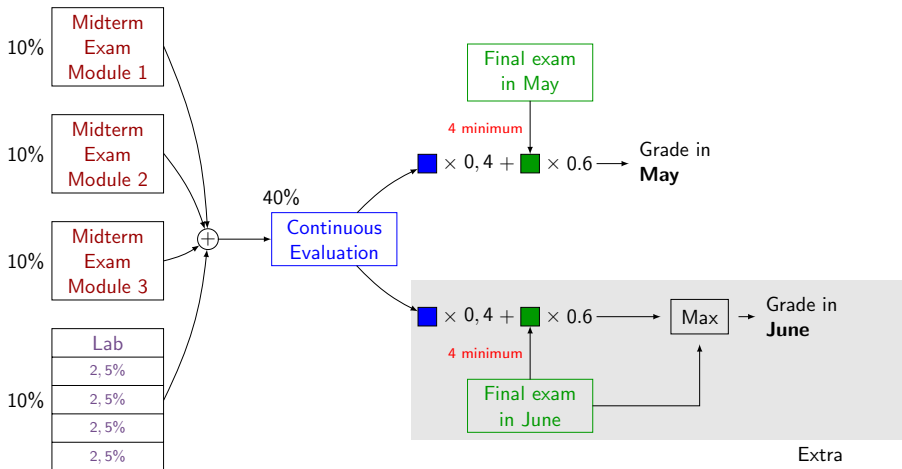
Communication Theory - 2020-2021

Regular sessions:

- Lectures in May, the 2nd and 5th. The 3rd has been cancelled (except for April the 29th from 12:00 to 12:30 (2020)).
- Exercises to be held in the form of the document.

Session	Topic	Date
1	1.1. Introduction	20.05.2021
2	1.2. Signal	27.05.2021
3	1.3. Modulation	03.06.2021
4	1.4. Multiplexing	10.06.2021
5	1.5. Error control	17.06.2021
6	1.6. Error control	24.06.2021
7	1.7. Error control	01.07.2021
8	1.8. Error control	08.07.2021
9	1.9. Error control	15.07.2021
10	1.10. Error control	22.07.2021
11	1.11. Error control	29.07.2021
12	1.12. Error control	05.08.2021
13	1.13. Error control	12.08.2021
14	1.14. Error control	19.08.2021
15	1.15. Error control	26.08.2021
16	1.16. Error control	02.09.2021
17	1.17. Error control	09.09.2021
18	1.18. Error control	16.09.2021
19	1.19. Error control	23.09.2021
20	1.20. Error control	30.09.2021
21	1.21. Error control	07.10.2021
22	1.22. Error control	14.10.2021
23	1.23. Error control	21.10.2021
24	1.24. Error control	28.10.2021
25	1.25. Error control	04.11.2021
26	1.26. Error control	11.11.2021
27	1.27. Error control	18.11.2021
28	1.28. Error control	25.11.2021
29	1.29. Error control	02.12.2021
30	1.30. Error control	09.12.2021
31	1.31. Error control	16.12.2021
32	1.32. Error control	23.12.2021
33	1.33. Error control	30.12.2021
34	1.34. Error control	06.01.2022
35	1.35. Error control	13.01.2022
36	1.36. Error control	20.01.2022
37	1.37. Error control	27.01.2022
38	1.38. Error control	03.02.2022
39	1.39. Error control	10.02.2022
40	1.40. Error control	17.02.2022
41	1.41. Error control	24.02.2022
42	1.42. Error control	03.03.2022
43	1.43. Error control	10.03.2022
44	1.44. Error control	17.03.2022
45	1.45. Error control	24.03.2022
46	1.46. Error control	31.03.2022
47	1.47. Error control	07.04.2022
48	1.48. Error control	14.04.2022
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100	1.100. Error control	13.04.2023

Evaluation of the course



Contents of the course

- 1 Noise in communications systems: stochastic processes, white noise, SNR

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Contents of the course

- 1 Noise in communications systems: stochastic processes, white noise, SNR
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- 3 Fundamental limits in communications
- 4 Analog modulations

What is the purpose of a communications system?

Goal: to transmit information between two points that are somehow connected by some **physical medium**

...the **physical medium** might be: a cable, the air, empty space...

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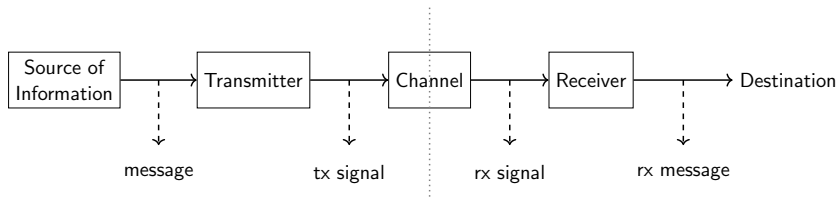
...the **physical medium** might be: a cable, the air, empty space...

Applications

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- peer-to-peer
- radio
- streaming
- ...plenty more

Block diagram

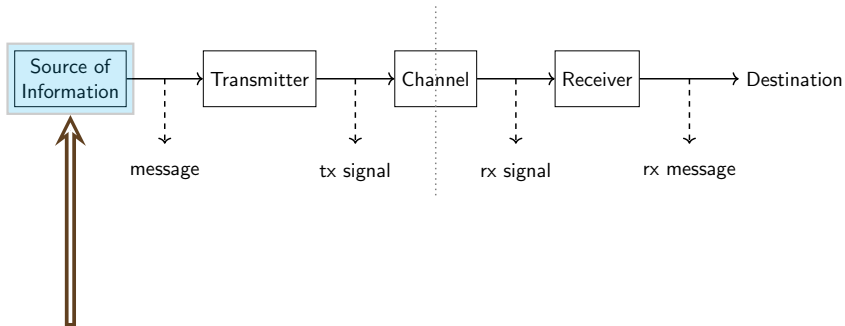
When focusing on the *functionality*, the structure of a typical communications system is:



message: physical manifestation of the information

We study each of the above blocks separately...

Source of information



Source of information

It aims a communicating/reporting something

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Messages produced might come in different formats

- voice
- text
- images
- ...

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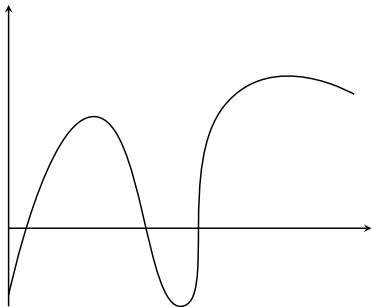
Sources can be

- analog
- digital

...according to the way in which information is represented

Analog source

It produces messages that are modeled as a continuous waveform.



This could represent variation in the air pressure, temperature variation, bitcoin price, price of stocks...

Digital source

It produces a sequence of *symbols* belonging to a **finite** set (the *alphabet*), each one sent during a certain time interval.



a symbol

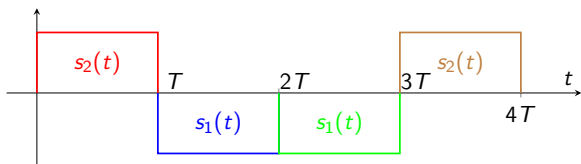
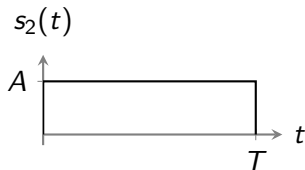
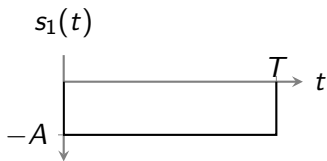
“a thing that represents or stands for something else”
(Oxford English Dictionary)

For us,

- a **symbol** translates into a (continuous-time) signal transmitted during a *symbol period* (usually denoted as T)
- the **alphabet** is a set of symbols

Digital source: examples I

Alphabet:



Digital source: examples II

More examples of alphabets

- {😊, 😞}

Digital source: examples II

More examples of alphabets

- $\{\text{😊}, \text{😞}\}$
- $\{A \sin(\omega_0 t), -A \sin(\omega_0 t)\}$

(the signals are *digital in amplitude*)

Digital source: examples II

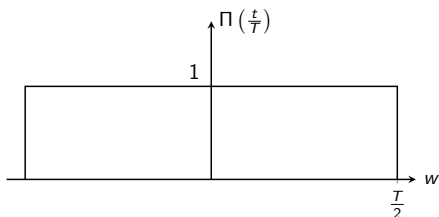
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- $\{\text{😊}, \text{😞}\}$
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(the signals are *digital in frequency*)

Digital source: examples II

More examples of alphabets

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- $\{A \sin(\omega_0 t), -A \sin(\omega_0 t)\}$
(the signals are *digital in amplitude*)
- $\{A \sin(\omega_1 t), A \sin(\omega_2 t)\}$
(the signals are *digital in frequency*)
- $\{A \Pi(\frac{t}{T}), -A \Pi(\frac{t}{T}), 3A \Pi(\frac{t}{T}), -3A \Pi(\frac{t}{T})\}$ where $\Pi(\frac{t}{T})$ is a rectangular pulse of length T centered at 0, i.e.,



(the signals are *digital in amplitude*)

Two different kinds of communication systems

type of source → type of communications system:

- **digital** source → digital communications system
examples: Fiber-optic communication (internet),
HDTV...pretty much everything
- **analog** source → analog communications system
examples: old TV, radio (for how long???)

how come we use digital communications system for nearly everything???

Two different kinds of communication systems

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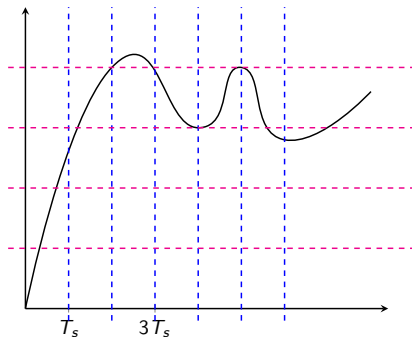
- **analog** source

→ analog communications system

→ digital communications system?

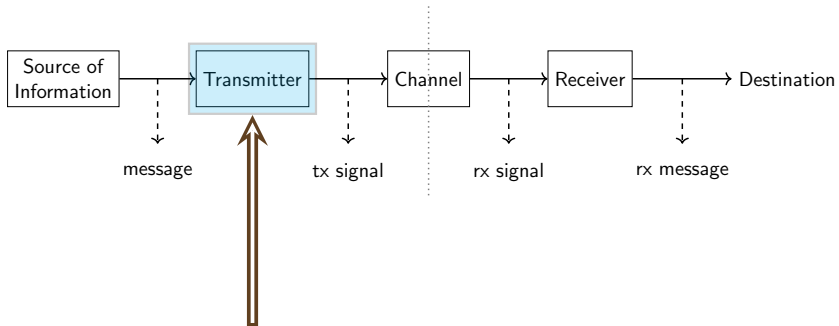
examples: old TV, radio (for how long??)

Digitizing signals



- **sampling** to discretize the time axis
 - no information loss if *Nyquist* condition holds
- **quantization** to discretize the amplitude
 - **information loss**

Transmitter



Transmitter

It shapes up the information coming from the source so that it can traverse the channel

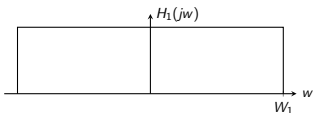
It needs to know whether the system is *analog* or *digital*...but also, whether the channel is

Transmitter

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- baseband...e.g.,

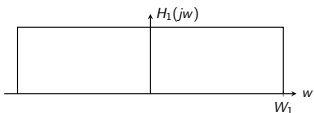


Transmitter

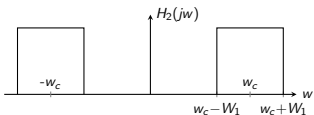
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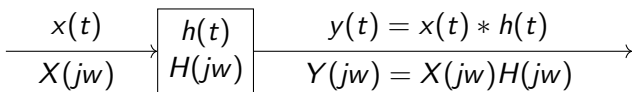
- passband...e.g.,



$$(\omega_c \gg W_1)$$

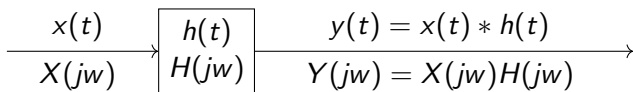
Transmission I

Here, we model the channel as an LTI system,

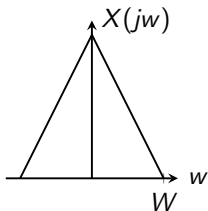


Transmission I

Here, we model the channel as an LTI system,



so, what happens if the spectrum of the signal to be transmitted is



Can the signal travel through both channels?

Transmission II

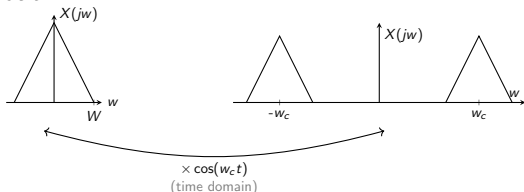
- $x(t)$ can travel through the baseband channel (*baseband* transmission)
 - without distortion if $W_1 > W$
 - **with** distortion if $W_1 < W$ (information loss)

Transmission II

- $x(t)$ can travel through the baseband channel (*baseband* transmission)
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- $x(t)$ **cannot** travel through the passband channel as it is, but...

Transmission II

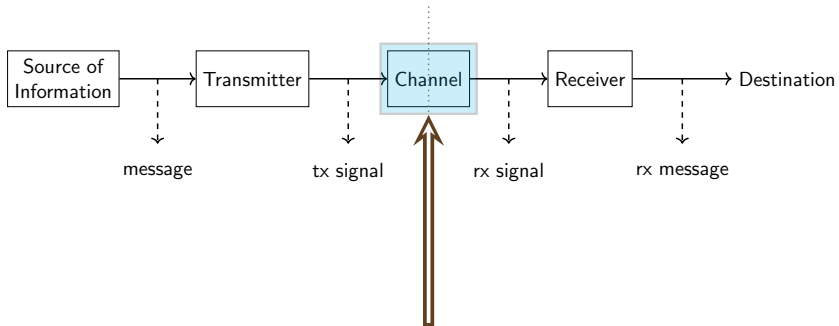
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 - **with** distortion if $W_1 < W$ (information loss)
- $x(t)$ **cannot** travel through the passband channel as it is, but...



...and we have *passband* transmission

The above operation is called modulation and $\cos(\omega_c t)$ is the so-called carrier signal

Channel



Channel

It is the physical medium through which information propagates

In general, it doesn't let the transmitted signal go through as it is:

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- disturbances
 - noise
 - interference

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- distortions due to the very own nature of the channel (modeled as an LTI system)

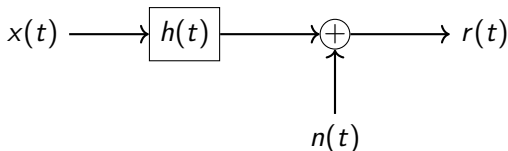
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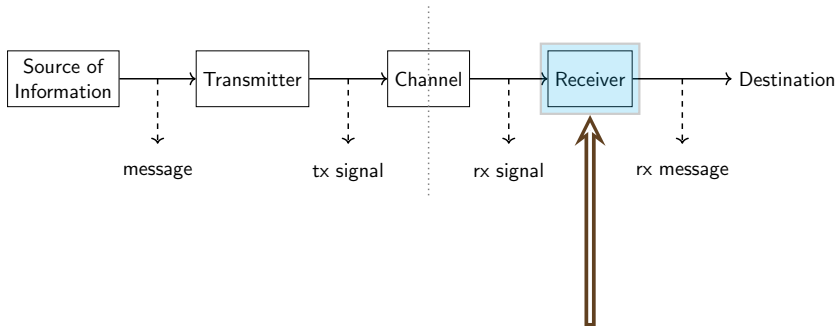
In general, it doesn't let the transmitted signal go through as it is:

- disturbances
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The channel is usually modeled like this:



Receiver



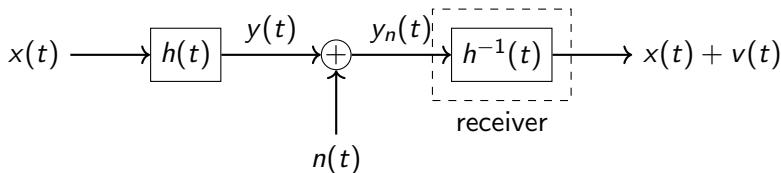
Receiver

It must recover the information transmitted as faithfully as possible

Among other things, it must

- 1 Demodulate, i.e., carry the signal back to its original frequency band
- 2 Reject disturbances
- 3 Fix channel distortions whenever possible

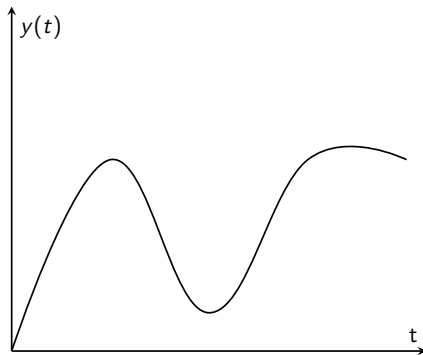
Ideally, we would like to find $h^{-1}(t)$ such that



Receiver in an analog system

2 and 3 are challenging in an analog system...

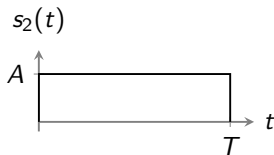
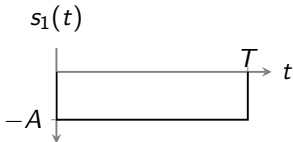
Let us assume we receive



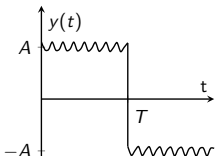
Was this the signal actually transmitted?

Receiver in an digital system

We know the alphabet of the system, e.g.,



If we receive...



- we **know** disturbances and/or distortions occurred
- we can *estimate* what was transmitted (making a *decision*)

This is the point of digital communication systems!!

Design of a system

When designing a system, we have to take into account (among other things):

- Quality
- Available technologies
- Cost
- Resources consumption

...we briefly review each one of them

Quality

We need a metric for the quality of a system so that the latter can be properly designed and compared against others

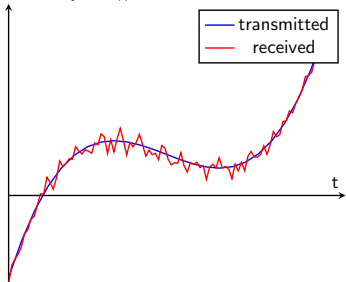
Different metrics for the two different kind of systems:

- analog system → fidelity
- digital system → error probability

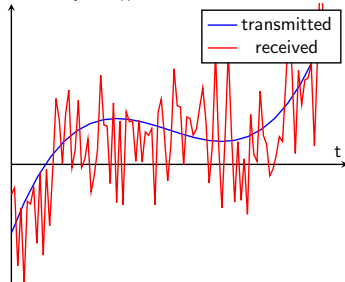
Quality in an analog system I

Fidelity refers to whether the received signal resembles the transmitted one.

Example #1



Example #2



On the left, the transmitted signal is *recognizable* in the received one...no so much on the right

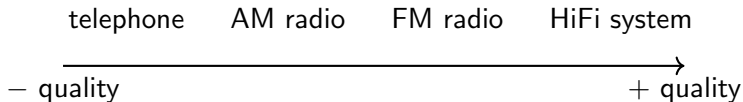
Quality in an analog system II

We need a *quantitative* measure of fidelity: it is the signal-to-noise ratio (SNR), which is defined as

S → power of the signal

N → power of the noise

Other parameter related to the quality: **bandwidth**



Quality in a digital system

We can count how many *symbols* were correctly received...and the **probability of error** is estimated as

$$P_e = \frac{\text{number of symbols incorrectly received}}{\text{overall number of symbols transmitted}}$$

Clearly,

- \uparrow quality \Rightarrow \downarrow probability of error (P_e)

Just like in analog systems, the **bandwidth** also has an impact here

- \uparrow bandwidth \Rightarrow \uparrow quality

Available technologies, Cost and Resources consumption

- before implementing a communications system, we should investigate the **available technologies**
 - is it worth it to use state-of-the-art technology? (how many people have access to it?)
 - an old (already deployed, cheap) technology might be fine for our purposes

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- we need to keep in mind the overall **cost** of the system...
 - how much is a terminal going to cost?
 - how much the base station?

Available technologies, Cost and Resources consumption

- before implementing a communications system, we should investigate the **available technologies**
 - is it worth it to use state-of-the-art technology? (how many people have access to it?)
 - an old (already deployed, cheap) technology might be fine for our purposes
- we need to keep in mind the overall **cost** of the system...
 - how much is a terminal going to cost?
 - how much the base station?
- **resources** don't come for free
 - can we take up as much bandwidth as we like?
 - how much transmission power is too much? (health factors, other systems deployed in the same space)

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- **encryption**
- versatility: the same communications system can transmit any kind of information (ultimately, everything is bits!!)

Drawbacks of digital communication systems

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- almost every source of information is analog (not a problem in practice...)

The advantages trump the drawbacks