Discrete stochastic processes

Marcelino Lázaro, Manuel A. Vázquez

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- Cyclostationarity:
  - Mean: $\mu_X[n+N] = \mu_X[n]$
  - Autocorrelation: $R_X[n+k+N, n+N] = R_X[n+k, n]$

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  - Power

$$P_X = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_X(e^{j\omega})\, d\omega = \begin{cases} R_X[0], & X[n] \text{ WSS} \\ \tilde{R}_X[0], & X[n] \text{ cyclostationary} \end{cases}$$

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- Cross statistics

$$R_{YX}[k] = R_X[k] * h[k]$$