



# Analog modulations

## Communication Theory

Manuel A. Vázquez

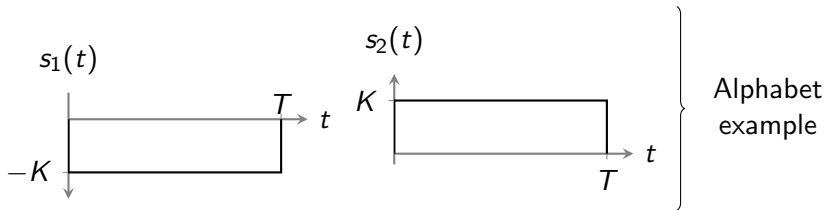
April 24, 2023

# Index

- 1 Introduction
  - Digital systems
  - Analog systems
  - Transmission
  - Baseband transmission
  - Passband transmission
  - Modulation
  - Linear vs. angular modulations
- 2 Linear modulations
  - Amplitude modulation
- 3 Angular modulations
  - Phase modulation (PM)
  - Frequency modulation (FM)

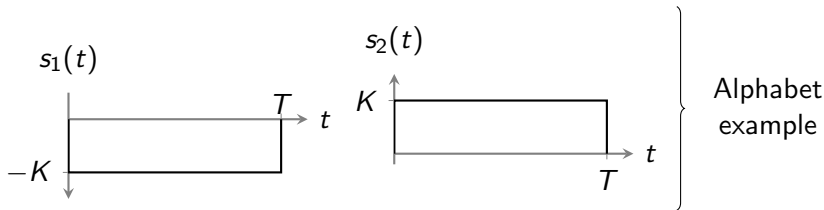
# Analog vs digital communications systems

In **digital** systems, information is carried by symbols in an alphabet,

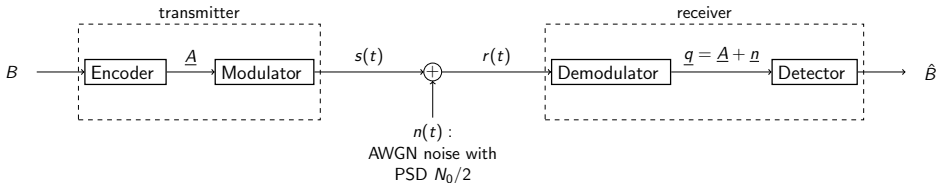


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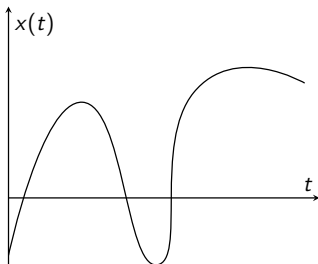


and the most basic scheme of a system is...



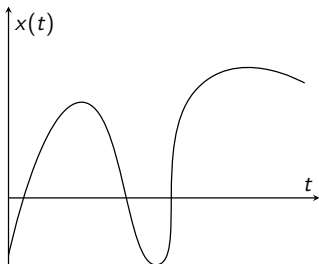
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In **analog** systems, information is contained in a continuous waveform...



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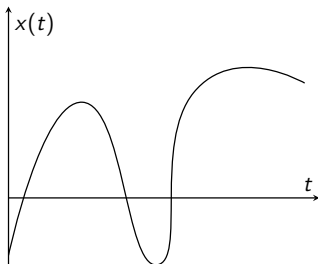
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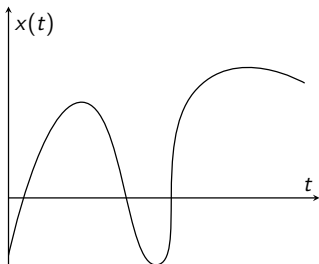


2 possibilities:

- discretize the signal and transmit it using a **digital** system

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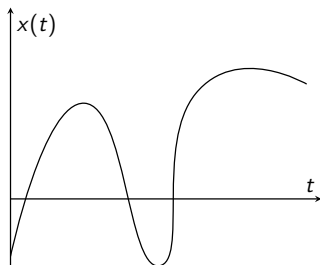
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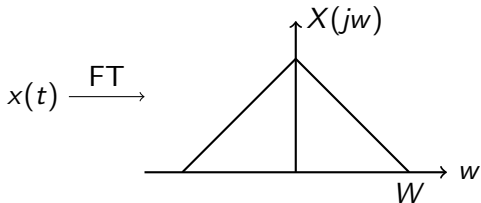
- discretize the signal and transmit it using a **digital** system
- transmit it directly using an **analog** system.

We focus on the latter approach...2 types of channels:

- **Baseband**
- **Passband**

# Analog transmission

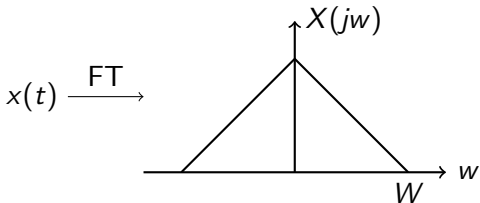
Assume we want to transmit



- baseband signal
- bandwidth  $W$

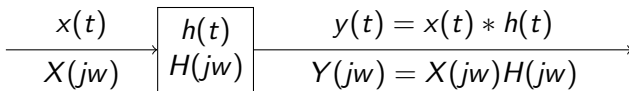
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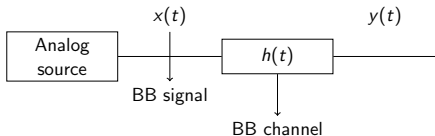
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...and we model the channel as a **Linear Time Invariant** system...



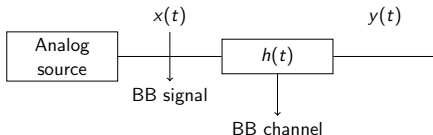
# Baseband transmission

The information signal is transmitted as is

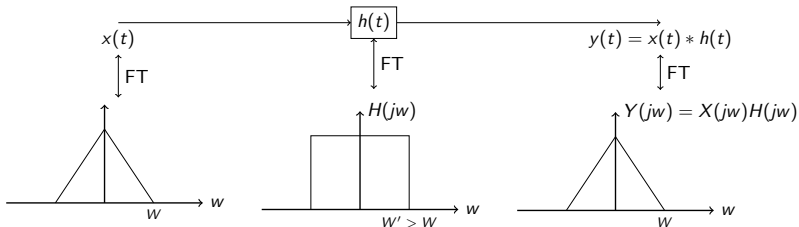


# Baseband transmission

The information signal is transmitted as is



Assuming an ideal **baseband** channel whose bandwidth is larger than that of the signal...



Examples: telephone subscriber loop, public address systems, closed-circuit TV

# Passband transmission

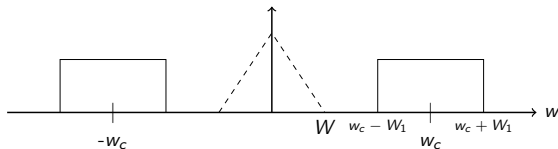
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- 2 center frequency,  $w_c$ , is **much** larger than the bandwidth of the signal.

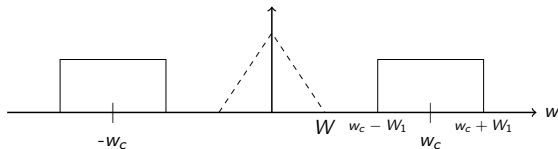


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- **passband** channel
- bandwidth  $2W_1$

- 2 entails  $w_c - W_1 > W$ , and hence

$$Y(jw) = X(jw)H(jw) = 0,$$

i.e., the signal **cannot** get through.

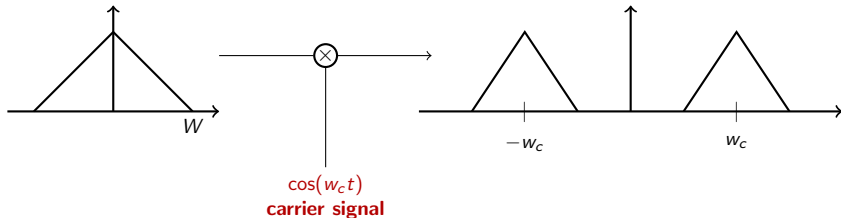


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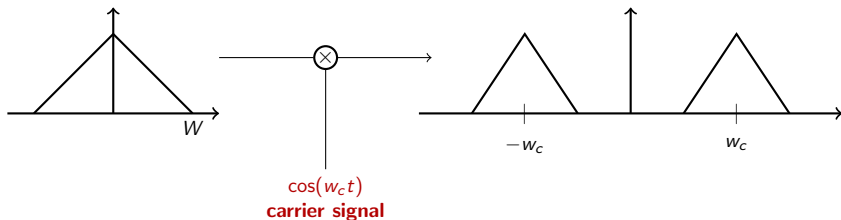


Mathematically,

$$x(t) \cos(\omega_c t) \xleftrightarrow{FT} \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

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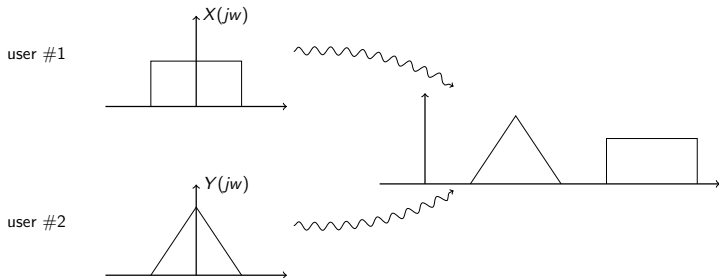
This operation is called **modulation**. It involves three signals

$$\underbrace{y(t)}_{\text{modulated signal}} = \underbrace{x(t)}_{\text{modulating signal}} \cdot \underbrace{\cos(\omega_c t)}_{\text{carrier signal}}$$

# Modulation

Other uses:

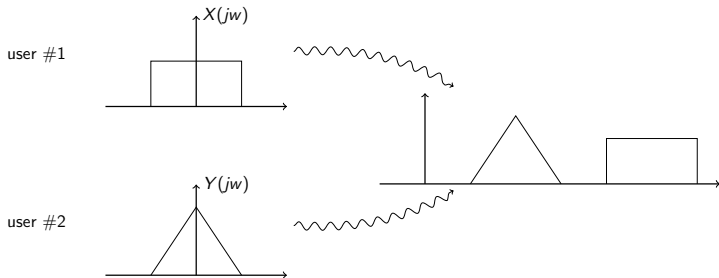
- multiuser systems



# Modulation

Other uses:

- multiuser systems



- protection against
  - noise
  - unauthorized users listening in the channel

# Types of modulation

Let us consider an information signal,  $x(t)$ , that is

- a **baseband** signal, i.e.,  $X(j\omega) = 0, \forall |\omega| > W$ 
  - a realization of a band-limited WSS random process  $X(t)$  with  $S_X(j\omega) = 0, \forall |\omega| > W$

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- a *power signal*, i.e., whose power is finite

Transmission of  $x(t)$  is achieved by embedding it in a **carrier** signal of the form

$$A_c \cos(\omega_c t + \phi_c)$$

amplitude                  frequency                  phase

and  $x(t)$  can be *modulating*

- the **amplitude**,
- the **frequency**, or
- the **phase**.



# Linear vs. angular modulations

In general, the modulated signal has the form

$$y(t) = r(t) \cos(w_c t + \varphi(t))$$

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- $x(t)$  is embedded in  $\varphi(t)$   $\rightarrow$  **angular modulation**

$$y(t) = A \cos(\omega_c t + \varphi(t))$$

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# Linear modulations

$$\begin{aligned}y(t) &= r(t) \cos(\omega_c t + \varphi) = r(t) (\cos(\omega_c t) \cos \varphi - \sin(\omega_c t) \sin \varphi) \\ &= \underbrace{r(t) \cos \varphi}_{x_i(t)} \cos(\omega_c t) - \underbrace{r(t) \sin \varphi}_{x_q(t)} \sin(\omega_c t),\end{aligned}$$

in-phase component

quadrature component

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$x_i(t)$  and  $x_q(t)$  are rewritten in a more convenient form

- $x_i(t) = r(t) \cos \varphi = A_c + A_m x(t)$
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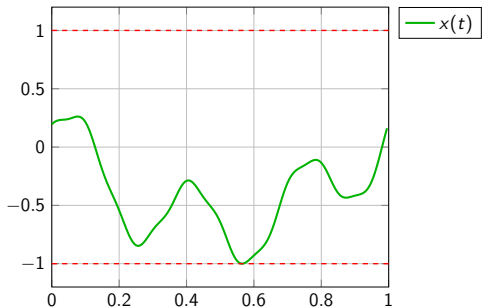
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# AM modulation

$$y(t) = (A_c + A_m x(t)) \cos(w_c t)$$

Let us assume  $|x(t)| \leq 1$  (if not, we can do  $x_n(t) = \frac{x(t)}{\max|x(t)|}$ , )



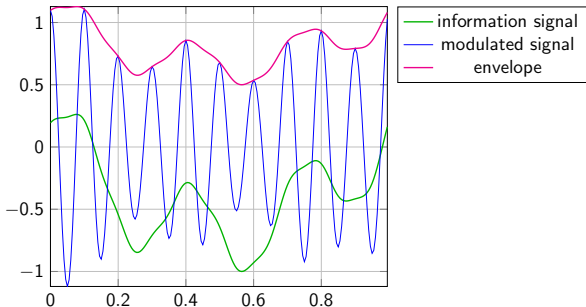
$$y(t) = \left( A_c + \frac{A_c}{A_c} A_m x(t) \right) \cos(w_c t) = A_c (1 + m x(t)) \cos(w_c t)$$

with

$$m = \frac{A_m}{A_c} \equiv \text{modulation index}$$

# AM demodulation

$A_c (1 + mx(t)) \geq 0$  implies this signal matches the **envelope** of the **modulated signal**,  $y(t)$



## Envelope of a signal

The **envelope** of an oscillating signal (for example, a cosine) is a *smooth* signal that outlines its extremes.

# AM demodulation

In order to guarantee  $A_c (1 + mx(t)) \geq 0$ , and assuming  $|x(t)| \leq 1$  (i.e. it is *normalized*), it is enough to choose

$$0 < m \leq 1.$$

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Indeed,

$$\left. \begin{array}{l} 0 < m \leq 1 \\ |x(t)| \leq 1 \end{array} \right\} \Rightarrow (1 + mx(t)) \geq 0 \Rightarrow \text{demodulation is very easy}$$

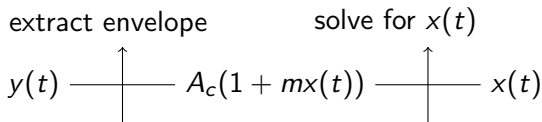
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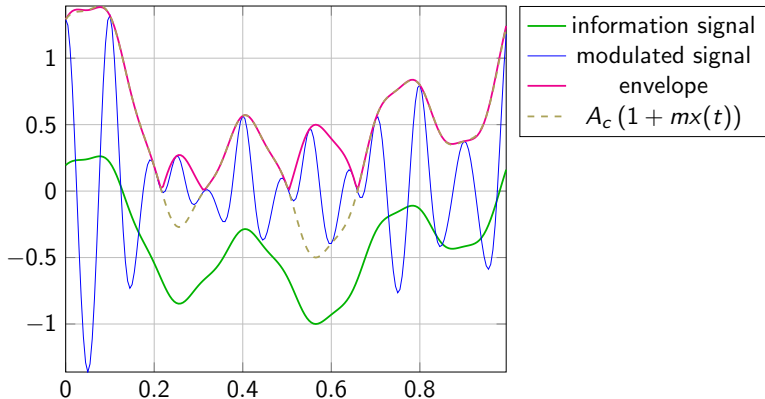
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# Overmodulation

If  $m > 1$ , it might happen that  $A_c(1 + mx(t)) < 0$  at some point...



The envelope is **not**  $A_c(1 + mx(t))$  anymore, and hence demodulation is not so easy. It is called **overmodulation**.

# Angular modulations

The information is in the argument of the **carrier**

## Drawbacks

- they are way **more complex** than linear modulations
  - sometimes, they need to be studied through approximations
- they take up **more bandwidth**



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## Advantages

- they are **less affected by noise...**
  - they trade off bandwidth for immunity against noise

# Algebra for Angular modulations

$$y(t) = A \cos(w_c t + \varphi(t)) = A \cos \phi(t),$$

where

$$\phi(t) = w_c t + \varphi(t) \equiv \textit{instantaneous phase}$$

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From it, we have

$$\frac{d\phi(t)}{dt} = w_i(t) \frac{\text{rad}}{\text{second}} \equiv \textit{instantaneous frequency},$$

in radians,

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$$\frac{1}{2\pi} \frac{d\phi(t)}{dt} = f_i(t) \text{ Hz} \equiv \textit{instantaneous frequency}.$$

in hertz.

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Doing some algebra

$$f_i(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} = \frac{1}{2\pi} \frac{d(w_c t + \varphi(t))}{dt} \stackrel{w_c = 2\pi f_c}{=} f_c + \frac{1}{2\pi} \frac{d\varphi(t)}{dt}$$

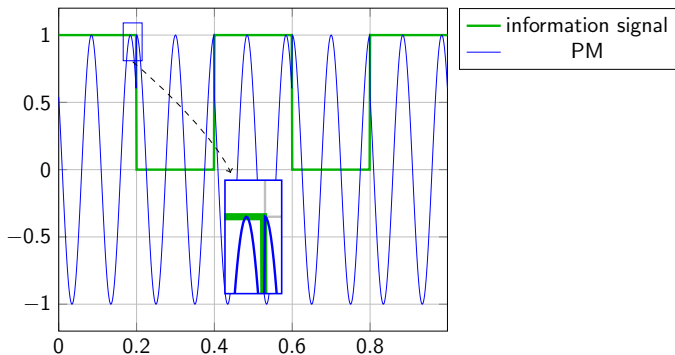
# Phase modulation

Being  $x(t)$  the information signal...

$$\varphi(t) = \beta x(t) \longrightarrow \text{Phase modulation (PM)}$$

with  $\beta \equiv$  phase deviation constant. Hence,

$$y(t) = A \cos(\omega_c t + \beta x(t))$$



# Frequency modulation

$$f_i(t) = f_c + f_d x(t) \longrightarrow \text{Frequency modulation (FM)}$$

with  $f_d \equiv$  frequency deviation.



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and hence

$$y(t) = A \cos \left( \omega_c t + 2\pi f_d \int_{-\infty}^t x(u) du \right)$$

