Modulation and detection

Manuel A. Vázquez

March 20, 2024

Index

Connection between bits and symbols

$M \equiv$ number of elements in the constellation $(\underline{a}_1, \underline{a}_2, \cdots, \underline{a}_M)$

- The **number of bits per symbol** is

$$m = \log_2 M$$

$E_s \equiv$ (mean) energy of the constellation

- The (mean) **bit energy** is defined as

$$E_b = \frac{E_s}{m}$$

$P_e \equiv$ probability of *symbol* error $\left(\frac{\#\text{ erroneous symbols}}{\#\text{ symbols transmitted}} = \frac{v}{w}\right)$

- Bit Error Rate (**BER**)

$$\left.\begin{array}{ll} \text{worst-case scenario} \rightarrow & BER = \frac{v \times m}{w \times m} = P_e \\ \text{best-case scenario} \rightarrow & BER = \frac{v \times 1}{w \times m} = \frac{P_e}{m} \end{array}\right\} \Rightarrow \frac{P_e}{m} \leq BER \leq P_e$$

Gray mapping

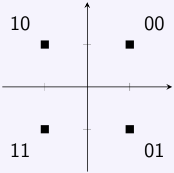
...a way to *induce* the best-case scenario

Premise

When an error happens we usually mistake a symbol for one of the adjacent ones.

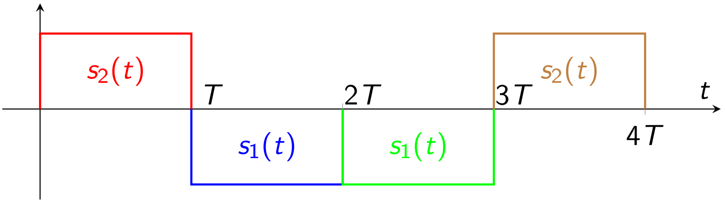
Gray mapping: assign sequences of bits that only differ in one bit to adjacent elements in the constellation

Example: $M = 4, N = 2$



It is the **optimal** way of assigning sequences of bits to symbols.

Transmission of a sequence of symbols



$T \equiv$ symbol period

$$R_s = \frac{1}{T} \equiv \textbf{ symbol rate } \left(\frac{\text{symbols}}{\text{second}} \text{ or } \textit{bauds}\right)$$

$m \equiv$ number of bits per symbol

$$R_b = m \cdot R_s \equiv \textbf{ bit rate } \left(\frac{\text{bits}}{\text{second}}\right)$$