Communications Theory

Introduction

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(Theory + Exercises)

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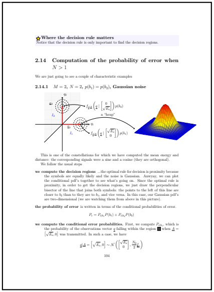
(Lab sessions)

Material

To be found in Aula Global:

- *Ad-hoc* notes



Where the decision rule matters
Notice that the decision rule is only important to find the decision regions.

2.14 Computation of the probability of error when $N > 1$

We are just going to see a couple of characteristic examples

2.14.1 $M = 2$, $N = 2$, $p(b_1) = p(b_2)$, Gaussian noise

This is one of the constellations for which we have computed the mean energy and distance: the corresponding signals were a sine and a cosine (they are orthogonal).

We follow the usual steps

we compute the decision regions ...the optimal rule for decision is proximity because the symbols are equally likely and the noise is Gaussian. Anyway, we can plot the conditional pdf's together to see what's going on. Since the optimal rule is proximity, in order to get the decision regions, we just draw the perpendicular bisector of the line that joins both symbols: the points to the left of this line are closer to b_2 than to they are to b_1, and vice versa. In this case, our Gaussian pdf's are two-dimensional (we are watching them from above in this picture).

the probability of error is written in terms of the conditional probabilities of error.

$$P_e = P_{e|b_1} P(b_1) + P_{e|b_2} P(b_2)$$

we compute the conditional error probabilities. First, we compute $P_{e|b_1}$, which is the probability of the observations vector $\underline{q}$ falling within the region ▩ when $\underline{A} = [\sqrt{E_s}, 0]$ was transmitted. In such a case, we have

$$\underline{q}|\underline{A} = [\sqrt{E_s}, 0] \sim \mathcal{N}\left(\begin{bmatrix} \sqrt{E_s} \\ 0 \end{bmatrix}, \frac{N_0}{2} \underline{I}_2\right)$$

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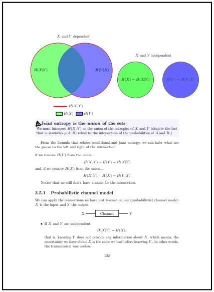

Joint entropy is the *union* of the sets
We must interpret $H(X, Y)$ as the union of the entropies of X and Y (despite the fact that in statistics $p(A, B)$ refers to the intersection of the probabilities of A and B.)

From the formula that relates conditional and joint entropy, we can infer what are the pieces to the left and right of the intersection:

if we remove $H(Y)$ from the union...

$$H(X, Y) - H(Y) = H(X|Y)$$

and, if we remove $H(X)$ from the union...

$$H(X, Y) - H(X) = H(Y|X)$$

Notice that we still don't have a name for the intersection.

3.5.1 Probabilistic channel model

We can apply the connections we have just learned on our (probabilistic) channel model: X is the input and Y the output

- If X and Y are independent

$$H(X|Y) = H(X),$$

that is, knowing Y does not provide any information about X, which means, the uncertainty we have about X is the same we had before knowing Y. In other words, the transmission was useless.

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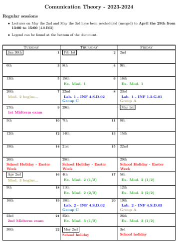
- Slides

Planning of the course



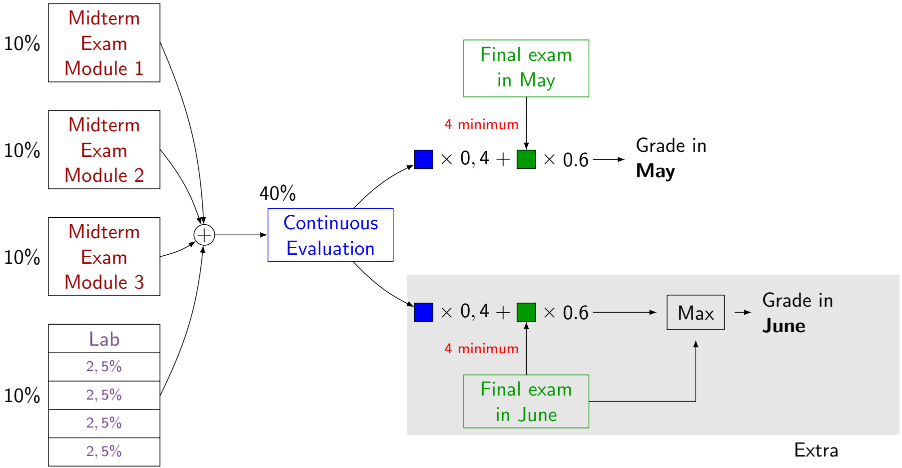
Comunication Theory - 2023-2024

Regular sessions

- Lectures on May the 2nd and May the 3rd have been rescheduled (merged) to **April the 29th from 13:00 to 15:00** (4.0.E03)
- Legend can be found at the bottom of the document.

Tuesday	Thursday	Friday
Jan 30th 1	Feb 1st 2	2nd
6th 3	8th 4	9th
13th 5	15th 6 Ex. Mod. 1	16th Ex. Mod. 1
20th 7 Mod. 2 begins...	22nd 8 Lab. 1 - INF 4.S.D.02 Group C	23rd Lab. 1 - INF 1.2.G.01 Group A
27th 9 1st Midterm exam	29th	Mar 1st
5th 10	7th 11	8th
12th 12	14th 13	15th
19th 14	21st 15	22nd
26th School Holiday - Easter Week	28th School Holiday - Easter Week	29th School Holiday - Easter Week
Apr 2nd 16 Mod. 3 begins...	4th 17 Ex. Mod. 2 (1/2)	5th Ex. Mod. 2 (1/2)
9th 18	11th Ex. Mod. 2 (2/2)	12th Ex. Mod. 2 (2/2)
16th 19	18th 20 Lab. 2 - INF 4.S.D.02 Group C	19th Lab. 2 - INF 4.S.D.03 Group A
23rd 21 2nd Midterm exam	25th Ex. Mod. 3 (1/2)	26th Ex. Mod. 3 (1/2)
30th 22	May 2nd School holiday	3rd School holiday

Evaluation of the course



Contents of the course

1. Noise in communications systems: stochastic processes, white noise, SNR
2. Modulation and detection in Gaussian channels: information modulation, demodulation and detection, error probability
3. Fundamental limits in communications
4. Analog modulations

What is the purpose of a communications system?

Goal: to transmit information between two points that are somehow connected by some physical medium

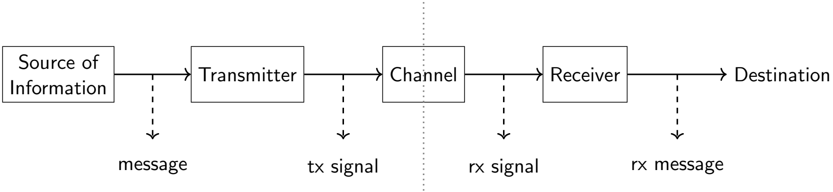
...the physical medium might be: a cable, the air, empty space...

Applications

- cellphone - base station
- base station - TV
- peer-to-peer
- radio
- streaming
- ...plenty more

Block diagram

When focusing on the *functionality*, the structure of a typical communications system is:



message: physical manifestation of the information

We study each of the above blocks separately...

Source of information

It aims a communicating/reporting something

Messages produced might come in different formats

- voice
- text
- images
- ...

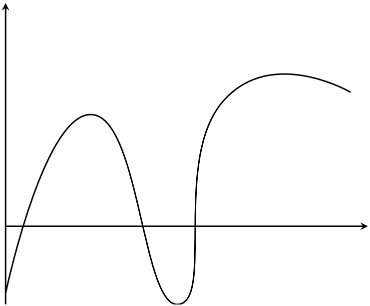
Sources can be

- analog
- digital

...according to the way in which information is represented

Analog source

It produces messages that are modeled as a continuous waveform.



This could represent variation in the air pressure, temperature variation, bitcoin price, price of stocks...

Digital source

It produces a sequence of *symbols* belonging to a **finite** set (the *alphabet*), each one sent during a certain time interval.

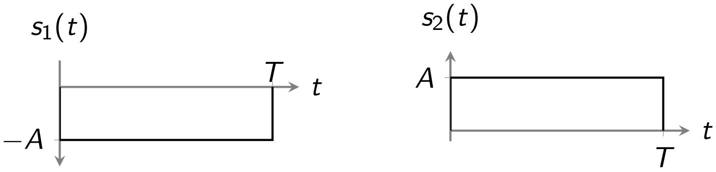
a symbol
"a thing that represents or stands for something else"
(Oxford English Dictionary)

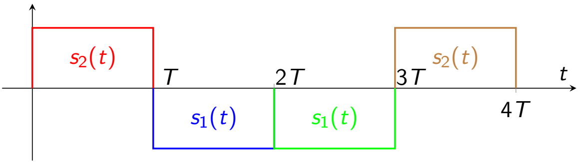
For us,

- a **symbol** translates into a (continuous-time) signal transmitted during a *symbol period* (usually denoted as T)
- the **alphabet** is a set of symbols

Digital source: examples I

Alphabet:





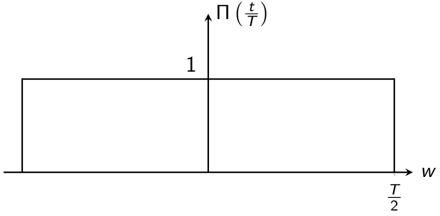
Digital source: examples II

More examples of alphabets

- {☺, ☹}
- $\{A\sin(w_0 t), -A\sin(w_0 t)\}$

  (the signals are *digital in amplitude*)
- $\{A\sin(w_1 t), A\sin(w_2 t)\}$

  (the signals are *digital in frequency*)
- $\left\{A\Pi\left(\frac{t}{T}\right), -A\Pi\left(\frac{t}{T}\right), 3A\Pi\left(\frac{t}{T}\right), -3A\Pi\left(\frac{t}{T}\right)\right\}$ where $\Pi\left(\frac{t}{T}\right)$ is a rectangular pulse of length T centered at 0, i.e.,



(the signals are *digital in amplitude*)

Two different kinds of communication systems

type of source → type of communications system:

- **digital** source → digital communications system

  examples: Fiber-optic communication (internet), HDTV...pretty much everything

- **analog** source → analog communications system

- **analog** source

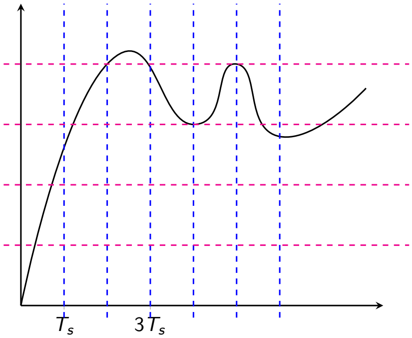
  → analog communications system

  → digital communications system?

  examples: old TV, radio (for how long??)

how come we use digital communications system for nearly everything???

Digitizing signals



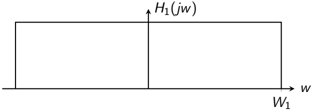
- **sampling** to discretize the time axis
  - no information loss if *Nyquist* condition holds
- **quantization** to discretize the amplitude
  - information loss

Transmitter

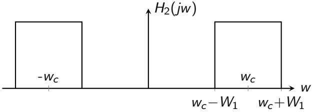
It shapes up the information coming from the source so that it can traverse the channel

It needs to know whether the system is *analog* or *digital*...but also, whether the channel is

- baseband...e.g.,



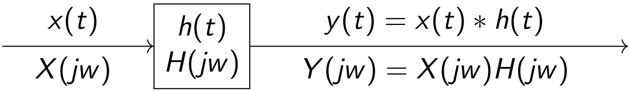
- passband...e.g.,



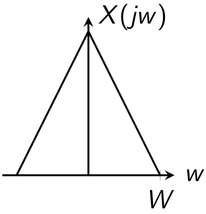
$(w_c \gg W_1)$

Transmission I

Here, we model the channel as an LTI system,



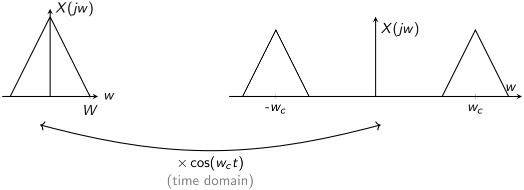
so, what happens if the spectrum of the signal to be transmitted is



Can the signal travel through both channels?

Transmission II

- $x(t)$ can travel through the baseband channel (*baseband* transmission)
  - without distortion if $W_1 > W$
  - **with** distortion if $W_1 < W$ (information loss)
- $x(t)$ **cannot** travel through the passband channel as it is, but...



...and we have *passband* transmission

The above operation is called modulation and $\cos(w_c t)$ is the so-called carrier signal

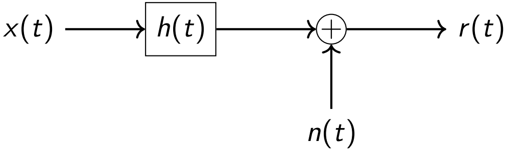
Channel

It is the physical medium through which information propagates

In general, it doesn't let the transmitted signal go through as it is:

- disturbances
  - noise
  - interference
- distortions due to the very own nature of the channel (modeled as an LTI system)

The channel is usually modeled like this:



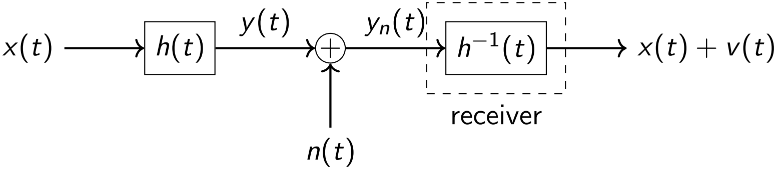
Receiver

It must recover the information transmitted as faithfully as possible

Among other things, it must

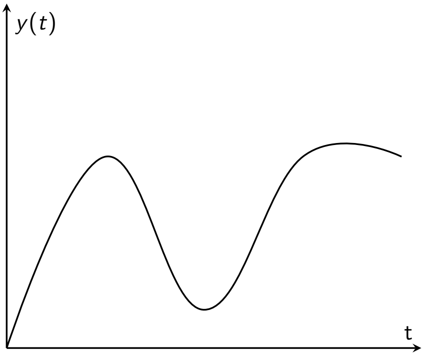
1. Demodulate, i.e., carry the signal back to its original frequency band
2. Reject disturbances
3. Fix channel distortions whenever possible

Ideally, we would like to find $h^{-1}(t)$ such that



Receiver in an analog system

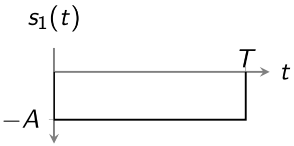
2 and 3 are challenging in an analog system...
Let us assume we receive

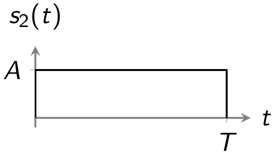


Was this the signal actually transmitted?

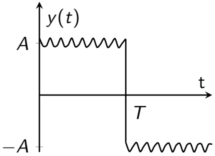
Receiver in an digital system

We know the alphabet of the system, e.g.,





If we receive...



- we **know** disturbances and/or distortions occurred
- we can *estimate* what was transmitted (making a *decision*)

This is the point of digital communication systems!!

Design of a system

When designing a system, we have to take into account (among other things):

- Quality
- Available technologies
- Cost
- Resources consumption

...we briefly review each one of them

Quality

We need a metric for the quality of a system so that the latter can be properly designed and compared against others

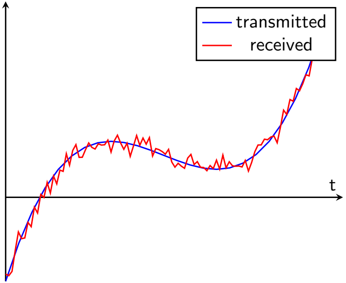
Different metrics for the two different kind of systems:

- analog system $\rightarrow$ fidelity
- digital system $\rightarrow$ error probability

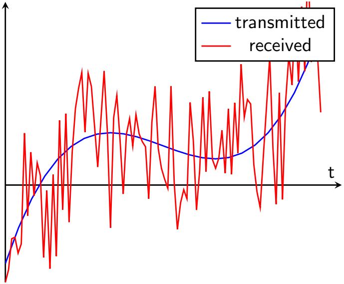
Quality in an analog system I

Fidelity refers to whether the received signal resembles the transmitted one.

Example #1



Example #2



On the left, the transmitted signal is *recognizable* in the received one...no so much on the right

Quality in an analog system II

We need a *quantitative* measure of fidelity: it is the signal-to-noise ratio (SNR), which is defined as

$$\frac{\mathbf{S}}{\mathbf{N}} \quad \begin{array}{l} \rightarrow \text{ power of the signal} \\ \rightarrow \text{ power of the noise} \end{array}$$

Other parameter related to the quality: **bandwidth**

telephone AM radio FM radio HiFi system

$-$ quality $\longrightarrow$ $+$ quality

Quality in a digital system

We can count how many *symbols* were correctly received...and the **probability of error** is estimated as

$$P_e = \frac{\text{number of symbols incorrectly received}}{\text{overall number of symbols transmitted}}$$

Clearly,

- $\uparrow$ quality $\Rightarrow \downarrow$ probability of error (P_e)

Just like in analog systems, the **bandwidth** also has an impact here

- $\uparrow$ bandwidth $\Rightarrow \uparrow$ quality

Available technologies, Cost and Resources consumption

- before implementing a communications system, we should investigate the **available technologies**
  - is it worth it to use state-of-the-art technology? (how many people have access to it?)
  - an old (already deployed, cheap) technology might be fine for our purposes
- we need to keep in mind the overall **cost** of the system...
  - how much is a terminal going to cost?
  - how much the base station?
- **resources** don't come for free
  - can we take up as much bandwidth as we like?
  - how much transmission power is too much? (health factors, other systems deployed in the same space)

Advantages of digital communication systems

- distortions and/or disturbances occurred during transmission can be detected and/or corrected
- there are error-detection and -correction schemes (**channel coding**)
- more reliable, flexible and cheaper circuits
- **encryption**
- versatility: the same communications system can transmit any kind of information (ultimately, everything is bits!!)

Drawbacks of digital communication systems

- synchronicity between transmitter and receiver is required
- a larger amount of bandwidth (expensive!!)
- almost every source of information is analog (not a problem in practice...)

The advantages trump the drawbacks