Discrete stochastic processes

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Discrete stochastic processes

- Notation: $X[n]$
- Statistical averages
  - Mean: $\mu_X[n] = E\left[X[n]\right]$
  - Autocorrelation: $R_X[n+k, n] = E\left[X[n+k]X[n]\right]$
- Wide Sense Stationarity (WSS):
  - Mean: $\mu_X[n] = \mu_X$
  - Autocorrelation: $R_X[n+k, n] = R_X[k]$
- Cyclostationarity:
  - Mean: $\mu_X[n+N] = \mu_X[n]$
  - Autocorrelation: $R_X[n+k+N, n+N] = R_X[n+k, n]$

Power spectral density and power

- Power spectral density
  - Wide-sense stationary processes

$$S_X(e^{j\omega}) = \mathcal{FT}\left\{R_X[k]\right\}$$

  - Cyclostationary processes

$$S_X(e^{j\omega}) = \mathcal{FT}\left\{\tilde{R}_X[k]\right\}, \qquad \tilde{R}_X[k] = \frac{1}{N}\sum_{n=0}^{N-1} R_X[n+k,n]$$

  - Power

$$P_X = \frac{1}{2\pi}\int_{-\pi}^{\pi} S_X(e^{j\omega})\, d\omega = \begin{cases} R_X[0], & X[n] \text{ WSS} \\ \tilde{R}_X[0], & X[n] \text{ cyclostationary} \end{cases}$$

LTI systems on **WSS** processes

- Mean of the output process

$$\mu_Y = \mu_X \sum_n h[n] = \mu_X H(0)$$

- Autocorrelation of the output process

$$R_Y[k] = R_X[k] * h[k] * h^*[-k]$$

- Power spectral density of the output process

$$S_Y(e^{j\omega}) = S_X(e^{j\omega}) \left| H(e^{j\omega}) \right|^2$$

- Cross statistics

$$R_{YX}[k] = R_X[k] * h[k]$$