Analog modulations

Communication Theory

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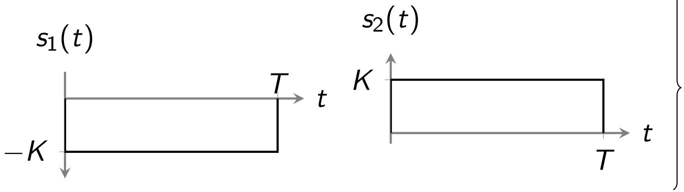
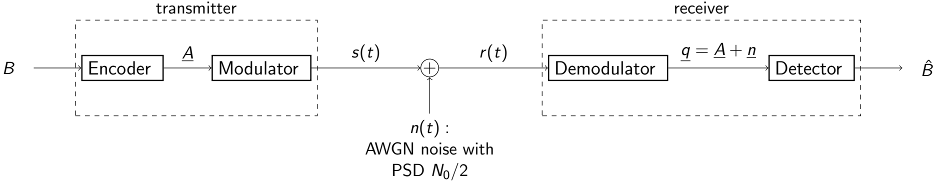
Analog vs digital communications systems

In digital systems, information is carried by symbols in an alphabet,



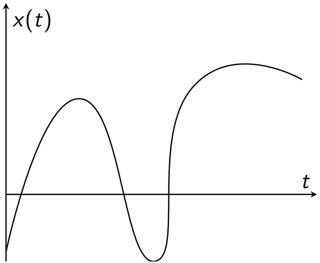
Alphabet example

and the most basic scheme of a system is...



Analog vs digital communications systems

In analog systems, information is contained in a continuous waveform...



2 possibilities:

- discretize the signal and transmit it using a digital system
- transmit it directly using an analog system.

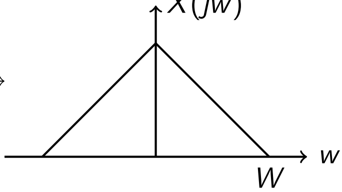
We focus on the latter approach...2 types of channels:

- Baseband
- Passband

Analog transmission

Assume we want to transmit

$$x(t) \xrightarrow{\text{FT}}$$



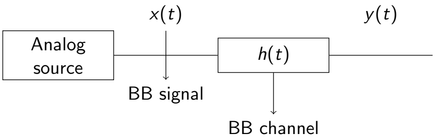
- baseband signal
- bandwidth W

...and we model the channel as a **L**inear **T**ime **I**nvariant system...

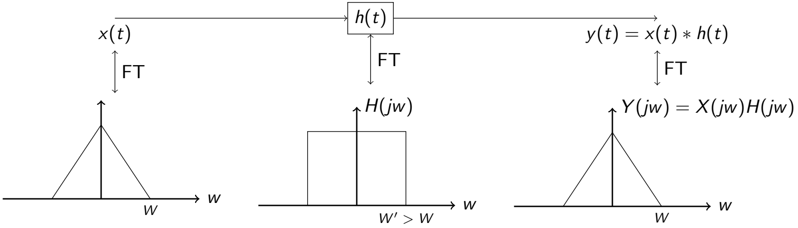
$$\xrightarrow[X(jw)]{x(t)} \boxed{\begin{matrix} h(t) \\ H(jw) \end{matrix}} \xrightarrow[Y(jw) = X(jw)H(jw)]{y(t) = x(t) * h(t)}$$

Baseband transmission

The information signal is transmitted as is



Assuming an ideal baseband channel whose bandwidth is larger than that of the signal...

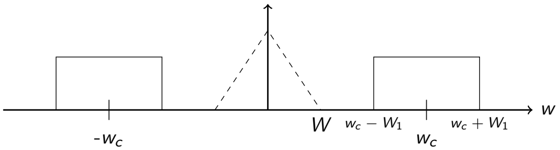


Examples: telephone subscriber loop, public address systems, closed-circuit TV

Passband transmission

Let us consider an ideal passband channel whose

1. bandwidth is larger than that of the signal, and whose
2. center frequency, w_c, is **much** larger than the bandwidth of the signal.



- passband channel
- bandwidth $2W_1$

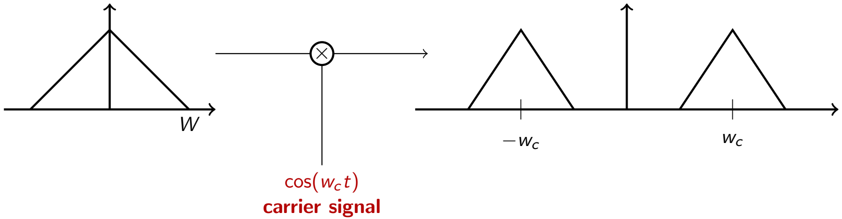
2 entails $w_c - W_1 > W$, and hence

$$Y(jw) = X(jw)H(jw) = 0,$$

i.e., the signal **cannot** get through.

Modulation

This is solved by shifting the spectrum of $x(t)$...



Mathematically,

$$x(t)\cos(w_c t) \xleftrightarrow{FT} \frac{1}{2}X(j(w - w_c)) + \frac{1}{2}X(j(w + w_c))$$

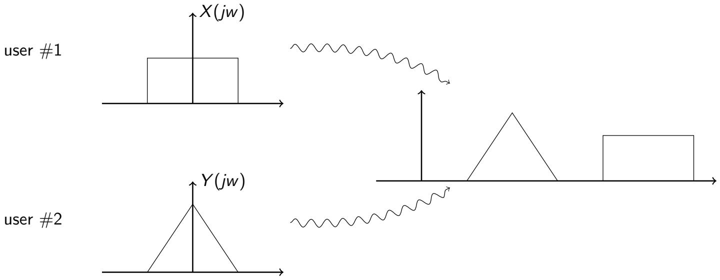
This operation is called **modulation**. It involves three signals

$$\underbrace{y(t)}_{\text{modulated signal}} = \underbrace{x(t)}_{\text{modulating signal}} \cdot \underbrace{\cos(\omega_c t)}_{\text{carrier signal}}$$

Modulation

Other uses:

- multiuser systems



- protection against
  - noise
  - unauthorized users listening in the channel

Types of modulation

Let us consider an information signal, $x(t)$, that is

- a baseband signal, i.e., $X(jw) = 0, \forall\, |w| > W$
  - a realization of a band-limited WSS random process $X(t)$ with $S_X(jw) = 0, \forall\, |w| > W$
- a *power signal*, i.e., whose power is finite

Transmission of $x(t)$ is achieved by embedding it in a carrier signal of the form

$$\underset{\text{amplitude}}{A_c} \cos(\underset{\text{frequency}}{w_c}\, t + \underset{\text{phase}}{\phi_c})$$

and $x(t)$ can be *modulating*

- the amplitude,
- the frequency, or
- the phase.

Linear vs. angular modulations

In general, the modulated signal has the form

$$y(t) = r(t)\cos(w_c t + \varphi(t))$$

- $x(t)$ is embedded in $r(t) \longrightarrow$ **linear** or **amplitude modulation**[1]

  $$y(t) = r(t)\cos(w_c t + \varphi)$$

  with *constant* frequency and phase.

- $x(t)$ is embedded in $\varphi(t) \longrightarrow$ **angular modulation**

  $$y(t) = A\cos(w_c t + \varphi(t))$$

  with *constant* amplitude and frequency.

---

[1] $\not\Rightarrow r(t) = x(t)$...since some transformation might be applied

Linear modulations

$$y(t) = r(t)\cos(w_c t + \varphi) = r(t)\left(\cos(w_c t)\cos\varphi - \sin(w_c t)\sin\varphi\right)$$
$$= \underbrace{r(t)\cos\varphi}_{x_i(t)} \cos(w_c t) - \underbrace{r(t)\sin\varphi}_{x_q(t)} \sin(w_c t),$$

in-phase component   quadrature component

$x_i(t)$ and $x_q(t)$ are rewritten in a more convenient form

- $x_i(t) = r(t)\cos\varphi = A_c + A_m x(t)$
- $x_q(t) = r(t)\sin\varphi = A_n \tilde{x}(t)$

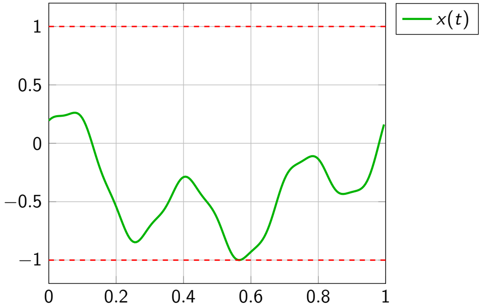
where $\tilde{x}(t)$ is a transformation of $x(t)$. The new parameters determine the type of modulation:

- $A_c \neq 0,\ A_m \neq 0,\ A_n = 0 \rightarrow$ AM modulation (*conventional* amplitude modulation)
- $A_c = 0,\ A_m \neq 0,\ A_n = 0 \rightarrow$ DSB modulation (Double Side Band modulation)
- $A_c = 0,\ A_m \neq 0,\ A_n \neq 0 \rightarrow$ SSB modulation (Single Side Band modulation)

AM modulation

$$y(t) = (A_c + A_m x(t))\cos(w_c t)$$

Let us assume $|x(t)| \leq 1$ (if not, we can do $x_n(t) = \frac{x(t)}{\max|x(t)|}$,)



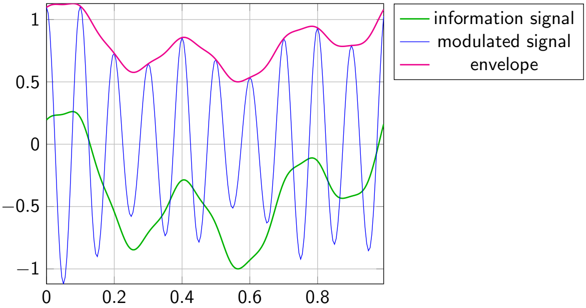
$$y(t) = \left(A_c + \frac{A_c}{A_c} A_m x(t)\right)\cos(w_c t) = A_c\,(1 + m x(t))\cos(w_c t)$$

with

$$m = \frac{A_m}{A_c} \equiv \textbf{modulation index}$$

AM demodulation

$A_c\,(1 + mx(t)) \geq 0$ implies this signal matches the envelope of the modulated signal, $y(t)$



Envelope of a signal

The **envelope** of an oscillating signal (for example, a cosine) is a *smooth* signal that outlines its extremes.

AM demodulation

In order to guarantee $A_c\,(1 + mx(t)) \geq 0$, and assuming $|x(t)| \leq 1$ (i.e. it is *normalized*), it is enough to choose

$$0 < m \leq 1.$$

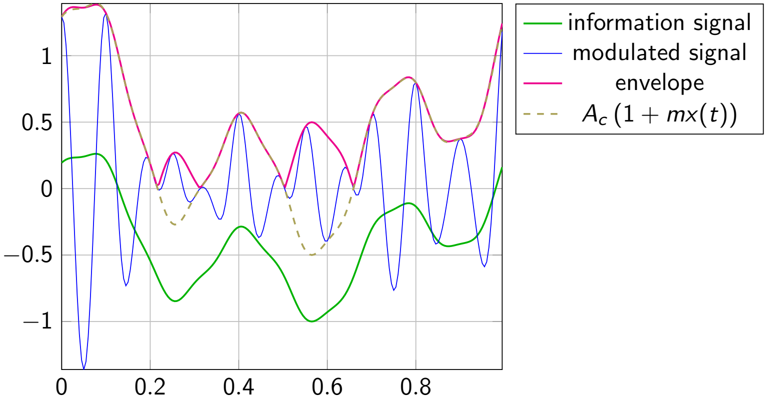
Indeed,

$$\left.\begin{array}{l} 0 < m \leq 1 \\ |x(t)| \leq 1 \end{array}\right\} \Rightarrow (1 + mx(t)) \geq 0 \Rightarrow \text{demodulation is very easy}$$

$$y(t) \xrightarrow{\text{extract envelope}} A_c(1 + mx(t)) \xrightarrow{\text{solve for } x(t)} x(t)$$

Overmodulation

If $m > 1$, it might happen that $A_c\,(1 + mx(t)) < 0$ at some point...



The envelope is **not** $A_c\,(1 + mx(t))$ anymore, and hence demodulation is not so easy. It is called **overmodulation**.

Angular modulations

The information is in the argument of the carrier

Drawbacks

- they are way **more complex** than linear modulations
  - sometimes, they need to be studied through approximations
- they take up **more bandwidth**

Advantages

- they are **less affected by noise**…
  - they trade off bandwidth for immunity against noise

Algebra for Angular modulations

$$y(t) = A\cos\left(w_c t + \varphi(t)\right) = A\cos\phi(t),$$

where

$$\phi(t) = w_c t + \varphi(t) \equiv \textit{instantaneous}\ \text{phase}$$

From it, we have

$$\frac{d\phi(t)}{dt} = w_i(t)\,\frac{\text{rad}}{\text{second}} \equiv \textit{instantaneous}\ \text{frequency},$$

in radians, which becomes

$$\frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_i(t)\,\text{Hz} \equiv \textit{instantaneous}\ \text{frequency}.$$

in hertz.

Doing some algebra

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{1}{2\pi}\frac{d\left(w_c t + \varphi(t)\right)}{dt} \overset{w_c = 2\pi f_c}{=} f_c + \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

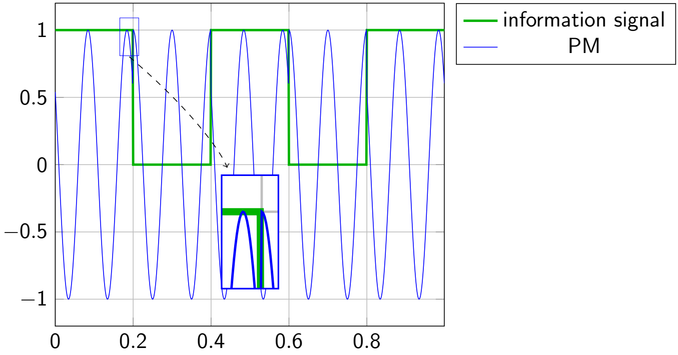
Phase modulation

Being $x(t)$ the information signal...

$$\varphi(t) = \beta x(t) \longrightarrow \text{Phase modulation (PM)}$$

with $\beta \equiv$ phase deviation constant. Hence,

$$y(t) = A\cos(w_c t + \beta x(t))$$



Frequency modulation

$$f_i(t) = f_c + f_d x(t) \longrightarrow \text{Frequency modulation (FM)}$$

with $f_d \equiv$ frequency deviation. It can be proved that

$$\varphi(t) = 2\pi f_d \int_{-\infty}^{t} x(u)du,$$

and hence

$$y(t) = A\cos\left(w_c t + 2\pi f_d \int_{-\infty}^{t} x(u)du\right)$$